# Self-Consistent-Field Calculation of the Geometry of Protonated Cyclopropane 

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#### Abstract

Accurate SCF-MO calculations performed on protonated cyclopropane, $\mathrm{C}_{3} \mathrm{H}_{7}+$, show that the ion is stable in the gas phase relative to $\mathrm{C}_{3} \mathrm{H}_{6}$ and that the equilibrium geometry corresponds to that of an edge-protonated species in which the proton is attached to the ring by a bridged hydrogen bond. Bonding is due principally to an interaction of the proton with in-plane carbon 2 p orbitals of the $3 \mathrm{e}^{\prime}$ molecular orbital of cyclopropane. This configuration is preferred over a face-protonated geometry involving bonding to the $2 \mathrm{p}_{2}$ orbitals above the ring. Electronic interaction is most favorable for carbon nuclei in a $60^{\circ}$ cyclopropane configuration, but the increase in nuclear repulsion due to the proton causes a ring opening to approximately $80^{\circ}$ for maximum stability.


Current interest in protonated cyclopropane, $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$, lies in areas of mass spectroscopy and reaction mechanism. In the former, it is well known that hydrocarbon spectra frequently show large amounts of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+},{ }^{2}$ a species which is thought to be exceptionally stable. Some question has existed concerning the structure of this ion. Since the appearance potential of $\mathrm{C}_{3} \mathrm{H}_{7}+$ in the spectra of $n$-alkanes and isoalkanes is the same, it was originally assumed that ions from both sources had the same isopropyl structure. ${ }^{3}$ A comparison of the ionization potential of the isopropyl radical with the appearance potential of the mass spectral $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$suggests that the two species are not the same, however, and, to resolve the anomaly, a protonated cyclopropane ring structure has been proposed for the ion. ${ }^{4}$ Such a structure also explains the presence of $\mathrm{CH}_{2} \mathrm{D}^{+}$in the spectra of propane-2- $d$, $n$-butane-$2-d$, and isobutane-2- $d$ as a result of a rearrangement of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$to give $\mathrm{CH}_{3}+$ and ethylene. ${ }^{5}$

Further evidence for the existence of protonated cyclopropane is found in studies of the solvolysis of cyclopropane in deuteriosulfuric acid ${ }^{6}$ and the deamination of $n$-propylamine ${ }^{7,8}$ where results indicate that both reactions proceed through the formation of a bridged hydrogen protonated cyclopropane intermediate. A reaction mechanism involving a similar bridged hydrogen structure also has been proposed by Hart ${ }^{9}$ to account for products occurring in the acylation of cyclopropane.

Theoretical investigations of the geometry of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$ thus far have been based on semiempirical techniques on an order of complexity comparable to Hückel theory. ${ }^{10}$ In addition to the well-known uncertainties
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inherent in such treatments, it has also been pointed out previously that ionic and charged molecules often show a behavior markedly different from that of neutral systems when the sum of near Hartree-Fock orbital energies is considered as a possible geometry determining quantity. ${ }^{11}$

Recently, it has become practicable to perform accurate $a b$ initio SCF calculations on systems the size of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$. In this study such calculations are reported for a number of possible geometries of the ion leading to a reasonably well-defined energy minimum. All electrons are included in the treatment and the basis set is sufficiently large to give results within 0.008 au of the Hartree-Fock energy for the carbon atom. ${ }^{12}$ Other applications of the techniques discussed here have previously been reported for a variety of polyatomic molecules. ${ }^{11-16}$ The results obtained in the study are interpreted by considering the effect of an additional proton on calculated molecular orbitals of cyclopropane. ${ }^{17}$ Charge density contour diagrams of certain molecular orbitals also are presented.

## Calculations

The basis set employed in this work has been described previously in an application to the ethylene molecule where numerical values of all exponents and coefficients are tabulated. ${ }^{12}$ Fixed groups, or linear combinations of Gaussian functions, are defined in terms of a fundamental basis set of single Gaussian functions, and lobe functions are used instead of spherical harmonic functions. Five-component p groups, plus three s groups, a three-component short range, a threecomponent long range, and a four-component intermediate range, are used for each carbon. A scaled five-component is function, $\eta=1.414$, is used for each hydrogen and the proton. Molecular orbitals are expanded as linear combinations of the 25 fixed basis groups, and coefficients are determined by a SCF
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Figure 1. Equilibrium configuration of the carbon atoms in cyclopropane. The angle $\theta$ is varied in studies of ring opening.
minimization of the total energy for each nuclear configuration.

In the first series of calculations, the nuclear framework of cyclopropane was held fixed at the experimental geometry shown in Figure 1, and the position of the proton was varied along the negative $y$ axis (edgeprotonated) and also along the $z$ axis (face-protonated). The total energy, nuclear repulsion energy, and sum of orbital energies are listed in Table I for each case examined. From these results it is evident that the edge-protonated form is strongly favored, giving a maximum binding energy of 0.197 au relative to $\mathrm{C}_{3} \mathrm{H}_{6}$ for proton coordinates of $(0,-2.90,0) .{ }^{18}$

Table I. Proton Coordinates ( $x, y, z$ ), Total Energy ( $E_{T}$ ), Sum of Orbital Energies ( $\Sigma \epsilon_{i}$ ), and Nuclear Repulsion Energy ( $V_{\mathbf{N}}$ ) for $60^{\circ} \mathrm{C}_{3} \mathrm{H}_{7}{ }^{+a}$

| $(x, y, z)$ | $E_{\mathrm{T}}$ | $\sum_{i} \epsilon_{i}$ | $V_{\mathrm{N}}$ |
| :---: | :---: | :---: | :---: |
| $(0,-1.43,0)$ | -116.76254 | -44.69126 | 87.19415 |
| $(0,-1.83,0)$ | -116.93411 | -44.55169 | 86.07143 |
| $(0,-2.40,0)$ | -117.08341 | -44.25937 | 84.51624 |
| $(0,-2.70,0)$ | -117.11009 | -44.08758 | 83.80051 |
| $(0,-2.90,0)$ | -117.11375 | -43.97244 | 83.36897 |
| $(0,-3.00,0)$ | -117.11239 | -43.91237 | 83.16640 |
| $(0,-3.25,0)$ | -117.10265 | -43.76558 | 82.69617 |
| $(0,0,1.00)$ | -116.78950 | -44.52664 | 86.87655 |
| $(0,0,2.00)$ | -116.95499 | -43.93338 | 84.42654 |
| $(0,0,2.75)$ | -116.96414 | -43.51049 | 82.96274 |
| $\left(\mathrm{C}_{3} \mathrm{H}_{6}\right)$ | -116.91636 |  | 75.73748 |

${ }^{a}$ Coordinates of carbon and hydrogen nuclei are given in Table II.

The possibility of ring opening is explored in a second series of calculations. The CCC angle $\theta$, shown in Figure 1, is increased successively to 80,100 , and $120^{\circ}$ with the relative geometry of $\mathrm{CH}_{2}$ groups held fixed; planes containing the nuclei in $\mathrm{CH}_{2}$ groups are taken to bisect the CCC angles to give over-all $\mathrm{C}_{2 \mathrm{v}}$ symmetry for each molecular geometry examined. Coordinates of all nuclei except the proton are given for each angle in Table II. Total energies, nuclear

[^0]

Figure 2. Valence orbital energy levels for $\mathrm{C}_{3} \mathrm{H}_{6}$ and $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$in their equilibrium geometries.
repulsion energies, and sums of orbital energies are given in Table III for each proton origin in the $80^{\circ}$ series, and in Table IV for the 100 and $120^{\circ}$ geometries, respectively. Minimum energy occurs in the $80^{\circ}$ case at proton coordinates $(0,-2.40,0)$ with a binding energy of 0.247 au with respect to cyclopropane.

Two additional calculations are reported in Table III for distorted geometries in which the proton is moved from its equilibrium position 0.5 au in the $x$ direction, $\mathrm{C}_{\mathrm{s}}(x y)$ symmetry, and 0.5 au in the $z$ direction, $\mathrm{C}_{\mathrm{s}}(y z)$ symmetry. In both cases the energy is increased although the increase is much more significant for distortion out of the plane of carbon nuclei.

## Bonding in Protonated Cyclopropane

Addition of a proton to the cyclopropane ring gives rise to two distinct effects. The first is a stabilization of all orbitals, as shown in Figure 2, caused by the presence of an additional attractive charge in the system. Inner-shell orbitals are omitted from the diagram, but these orbitals also exhibit the same dramatic stabilization in going from $\mathrm{C}_{3} \mathrm{H}_{6}$ to $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$.

The second effect concerns interaction of molecular orbitals of proper symmetry with the proton basis function to give $\mathrm{a}_{1}$ orbitals in the $\mathrm{C}_{2 \mathrm{v}}$ protonated form. In $60^{\circ}$ cyclopropane, the highest energy orbital of such symmetry is one of the doubly degenerate $3 \mathrm{e}^{\prime}$ molecular orbitals which forms a CC bond composed essentially of $2 \mathrm{p}_{x}$ and $2 \mathrm{p}_{y}$ atomic orbitals. Figure 3 shows the approximate composition of this orbital and also of the next lowest molecular orbital of $a_{1}$ symmetry, the $3 a_{1}{ }^{\prime}$. Analysis of the SCF vector calculated for the $80^{\circ}$ equilibrium configuration case shows the proton coefficient to be largest in the $3 \mathrm{e}^{\prime} \rightarrow 6 \mathrm{a}_{1}$ orbital with a value of 0.39 . In the $5 a_{1}, 4 a_{1}$, and $3 a_{1}$ orbitals, coefficients are smaller at $-0.11,0.15$, and 0.08 , respectively. A charge-density contour diagram of the $6 a_{1}$ molecular orbital is shown in Figure 4 illustrating the general features of the bridged hydrogen bond. A diagram for the $3 \mathrm{a}_{1}{ }^{\prime} \rightarrow 5 \mathrm{a}_{1}$ orbital shown in Figure 5

Table II. Coordinates of Carbon and Hydrogen Nuclei in $60,80,100$, and $120^{\circ}$ Cases of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$

|  | $60^{\circ}$ | $80^{\circ}$ | $100^{\circ}$ | $120^{\circ}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{C}_{1}$ | $(0,1.6628,0)$ | $(0,1.6628,0)$ | $(0,1.6628,0)$ | $(0,1.6628,0)$ |
| $\mathrm{H}_{\mathrm{a}}$ | $(0,2.6738, \pm 1.7511)$ | $(0,2.6728, \pm 1.7511)$ | $(0,2.6738 \pm 1.7511)$ | $(0,2.6738, \pm 1.7511)$ |
| $\mathrm{H}_{\mathrm{b}}$ | $(1.44,-0.8314,0)$ | $(1.8512,-0.5434,0)$ | $(2.2062,-0.1885,0)$ | $(2.4942,0.2228,0)$ |
| $\mathrm{C}_{2}$ |  |  |  |  |
| $\mathrm{H}_{\mathrm{a}}$ | $(2.3156,-1.3369, \pm 1.7511)$ | $(2.7675,-0.9707, \pm 1.7511)$ | $(3.1562,-0.5342, \pm 1.7511)$ | $(3.4707,-0.0389, \pm 1.7511)$ |
| $\mathrm{H}_{\mathrm{b}}$ | $(-1.44,-0.8314,0)$ | $(-1.8512,-0.5434,0)$ | $(-2.2062,-0.1885,0)$ | $(-2.4942,0.228,0)$ |
| $\mathrm{C}_{3}$ | $(-2.7675,-0.9707, \pm 1.7511)$ | $(-3.1562,-0.5342, \pm 1.7511)$ | $(-3.4707,-0.0389, \pm 1.7511)$ |  |
| $\mathrm{H}_{\mathrm{a}}$ | $(-2.3156,-1.3369, \pm 1.7511)$ | $(-2.7675$ |  |  |

Table III. Proton Coordinates, Total Energy, Sum of Orbital Energies, and Nuclear Repulsion Energy for $80^{\circ} \mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$in $\mathrm{C}_{2 \mathrm{v}}$ and $\mathrm{C}_{8}$ Symmetrya

| $(x, y, z)$ | $E_{\mathrm{T}}$ | $\sum_{i} \epsilon_{i}$ | $V_{\mathrm{N}}$ |
| :---: | :---: | :---: | :---: |
| $(0,-1.2,0)$ | -117.01855 | -44.14015 | 81.91237 |
| $(0,-1.4,0)$ | -117.06279 | -44.09570 | 81.52235 |
| $(0,-1.8,0)$ | -117.12967 | -43.99163 | 80.71469 |
| $(0,-2.2,0)$ | -117.16067 | -43.86282 | 79.93181 |
| $(0,-2.4,0)$ | -117.16353 | -43.79226 | 79.56418 |
| $(0,-2.8,0)$ | -117.15029 | -43.64080 | 78.88845 |
| $( \pm 0.5,-2.4,0)$ | -117.16159 | -43.79708 | 79.59890 |
| $(0,-2.4, \pm 0.5)$ | -117.10051 | -43.57264 | 79.46781 |

${ }^{\text {a }}$ Coordinates of carbon and hydrogen nuclei are given in Table II.
illustrates much less bonding tendency. The total charge density in the region of the bridged hydrogen bond is given in Figure 6.


Figure 3. General features of the $3 \mathrm{a}_{1}^{\prime} \rightarrow 5 \mathrm{a}_{1}$ and $3 \mathrm{e}^{\prime} \rightarrow 6 \mathrm{a}_{1}$ molecular orbitals of cyclopropane.

Results obtained by opening the $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$ring are shown in Figure 7 where energies of valence orbitals calculated at optimum proton positions are plotted as a function of CCC angle. Inner-shell molecular orbitals,

Table IV. Proton Coordinates, Total Energy, Sum of Orbital Energies, and Nuclear Repulsion Energy for 100 and $120^{\circ} \mathrm{C}_{3} \mathrm{H}_{7}+a$

| $(x, y, z)$ | $E_{\mathrm{T}}$ | $\Sigma_{i}$ | $V_{\mathrm{N}}$ |  |
| :--- | :---: | :---: | :---: | :---: |
| $100^{\circ}$ Geometry |  |  |  |  |
| $(0,-0.188,0)$ | -116.82370 | -44.08083 | 80.28613 |  |
| $(0,-0.40,0)$ | -116.88282 | -44.00468 | 79.90253 |  |
| $(0,-0.60,0)$ | -116.93052 | -43.93621 | 79.55195 |  |
| $(0,-0.80,0)$ | -116.97290 | -43.87477 | 79.20382 |  |
| $(0,-1.50,0)$ | -117.07368 | -43.69864 | 77.99156 |  |
| $(0,-1.80,0)$ | -117.09097 | -43.62765 | 77.49363 |  |
| $(0,-2.00,0)$ | -117.09441 | -43.57883 | 77.17590 |  |
| $(0,-2.20,0)$ | -117.09226 | -43.52779 | 76.87167 |  |
| $120^{\circ}$ Geometry |  |  |  |  |
| $(0,0.22,0)$ | -116.61101 | -43.98268 | 79.35998 |  |
| $(0,0,0)$ | -116.74150 | -43.89054 | 78.74675 |  |
| $(0,-0.20,0)$ | -116.81230 | -43.81074 | 78.27846 |  |
| $(0,-0.40,0)$ | -116.86052 | -43.73392 | 77.85803 |  |
| $(0,-1.00,0)$ | -116.94706 | -43.53708 | 76.75511 |  |
| $(0,-1.40,0)$ | -116.97520 | -43.43677 | 76.10549 |  |
| $(0,-1.80,0)$ | -116.98288 | -43.34769 | 75.51457 |  |
| $0,-2.20,0)$ | -116.97420 | -43.25774 | 74.98139 |  |

${ }^{a}$ Coordinates of carbon and hydrogen nuclei are given in Table II.
$1 a_{1}, 1 b_{2}$, and $2 a_{1}$, which are essentially linear combinations of carbon 1s orbitals, are not significantly affected by these angle variations. The lowest lying valence orbital is the $3 a_{1}$ consisting of long-range s groups on each carbon plus a smaller amount of $2 \mathrm{p}_{x}$ character from carbons 2 and 3. A change in angle from 60 to $120^{\circ}$ results in a greater $s$ contribution from $\mathrm{C}_{1}$ and the proton, but a decrease in interaction between the long-range $s$ and $2 p_{x}$ orbitals on $C_{2}$ and $C_{3}$ leads to a net decrease in stability.

The $2 \mathrm{~b}_{2}$ molecular orbital which is mainly an antisymmetric combination of long-range s groups on $\mathrm{C}_{2}$ and $C_{3}$ exhibits a stabilization with increased angle that can be attributed to a decrease in antibonding character with increasing distance between centers. The next orbital, $4 \mathrm{a}_{1}$, containing some proton character, shows a decrease in stability with increasing angle. This orbital is mainly a combination of long-range s, $2 \mathrm{p}_{x}$, and $2 \mathrm{p}_{y}$ functions representing bonds from $\mathrm{C}_{2}$ and $\mathrm{C}_{3}$ to $\mathrm{C}_{1}$; opening the ring tends to remove $2 \mathrm{p}_{\nu}$ atomic orbitals from bonding and leads to a decrease in stability.

Of the molecular orbitals which involve $2 \mathrm{p}_{z}$ functions, the $1 b_{1}$ and $2 b_{1}$ show a decrease in stability with increasing angle. The $1 b_{1}$, composed essentially of $2 p_{2}$ functions on $\mathrm{C}_{2}$ and $\mathrm{C}_{3}$, loses stability as the distance between the p functions increases while the decrease in stability of the $2 b_{1}$ is due to a weakening of the bond between the $2 p_{z}$ on $C_{1}$ with the two attached hydrogens, apparently in favor of increased CH bonding at


Figure 4. Charge density contour of the $6 \mathrm{a}_{1}$ molecular orbital of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$.


Figure 5. Charge density contour of the $5 a_{1}$ molecular orbital of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$.
centers $C_{2}$ and $C_{3}$. The $1 a_{2}$ molecular orbital is the third orbital involving $2 p_{z}$ functions and is stabilized by an increase in angle which increases the distance between the antibonding lobes on $\mathrm{C}_{2}$ and $\mathrm{C}_{3}$. In general, the trends in the three above orbitals are to be expected; however, the magnitudes of the changes in stability are not large and do not have a major effect in determining the geometry of the molecule.

The $5 \mathrm{a}_{1}$ orbital would seem to be rather important at first since it is high lying and of proper symmetry to accommodate the proton, but the destabilization of this orbital is no greater than that of any other orbital en-


Figure 6. Total charge density contour of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$in the region of the proton.


Figure 7. Valence orbital energies of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$as a function of CCC angle, $\theta$, with the proton in the position of maximum stability for each angle.
countered thus far. The orbital consists mainly of $2 \mathrm{p}_{x}$ and $2 \mathrm{p}_{y}$ functions as illustrated in Figure 3, and its decrease in stability is analogous to that of the $4 a_{1}$ except for an increase in proton contribution with increase in angle.

The $6 a_{1}$ is the highest molecular orbital which can interact with the proton basis function and in this respect is the most significant in determining the optimum angle of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$. As shown in Figure 7, the orbital undergoes a pronounced decrease in stability with increase in angle. This decrease can be attributed directly to a weakening of the bridged hydrogen bond caused by movement of carbon $p$ function away from the bonding region. The sharpness of the decrease in


Figure 8. Valence orbital energies of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$for equilibrium and distorted geometries.
stability effectively rules out both 100 and $120^{\circ}$ as possible optimum angles.

The highest molecular orbital, the $3 \mathrm{~b}_{2}$, is stabilized by an increase in angle. Essentially, this orbital is a combination of $2 p_{x}$ on $C_{1}$ and $2 p_{x}$ and $2 p_{y}$ functions on $\mathrm{C}_{2}$ and $\mathrm{C}_{3}$; increasing the angle causes stabilization by reducing the antibonding interaction between functions on centers 2 and 3.

Pursuing the orbital energy arguments outlined above would lead to a rationalization of an optimum angle of $60^{\circ}$ since it is for this geometry that the sum of orbital energies (and the sum of valence orbital energies) is lowest and the bridged hydrogen bond orbital, $6 a_{1}$, is most stable. The key point in understanding why in fact the $80^{\circ}$ configuration is favored lies in a consideration of nuclear and electronic repulsion. The total energy of the system which of course is the geometry determining quantity can be expressed as

$$
E=2 \sum_{i} \epsilon_{i}+\left(V_{\mathrm{n}}-E_{\mathrm{r}}\right)
$$

where the summation is over all doubly occupied molecular orbitals with energy $\epsilon_{i}$, and $V_{n}$ and $E_{\mathrm{r}}$ are nuclear and electronic repulsion terms, respectively. As has been pointed out previously, angular variations in nonionic systems frequently result in changes in $V_{n}$ sufficiently parallel to those in $E_{\mathrm{r}}$ such that $2 \Sigma \epsilon_{i}$ shows a minimum near that of $E$. In $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$, the nuclear repulsion increases by 3.8 au in going from 80 to $60^{\circ}$, but this increase is not completely offset by changes in $E_{\mathrm{r}}$ and as a result the calculated minimum $E$ occurs at $80^{\circ}$. This conclusion clearly illustrates the need for establishing numerically significant changes in $\epsilon_{i}$ before advancing geometry predictions based on orbital energies, and in addition obviously suggests a criticism of geometry prediction from semiempirical theories which do not explicitly include effects of nuclear repulsion.

As the proton is moved toward the ring from its equilibrium position, there is an increase in proton coefficients in the lower lying $a_{1}$ orbitals, $3 a_{1}, 4 a_{1}$, and $5 a_{1}$, at the expense of the $6 a_{1}$ as shown in Table $V$.


Figure 9. Valence orbital energies of face-protonated $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$as a function of protion $z$ coordinate.

This decrease in interaction with the molecular orbital that is best able to form a bridged-hydrogen bond, plus the effect of increased nuclear repulsion, is sufficient to establish an equilibrium configuration at somewhat greater distance from the ring than might be expected, at $(0,-2.40,0)$ au.

Table V. Proton Expansion Coefficients and Proton Coordinates for $\mathrm{a}_{1}$ Valence Molecular Orbitals of $80^{\circ} \mathrm{C}_{2 \mathrm{v}} \mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$

| $(x, y, z)$ | $3 \mathrm{a}_{1}$ | $4 \mathrm{a}_{1}$ | $5 \mathrm{a}_{1}$ | $6 \mathrm{a}_{1}$ |
| :---: | :---: | :---: | :---: | :---: |
| $(0,-1.2,0)$ | 0.241 | -0.209 | -0.268 | 0.308 |
| $(0,-1.4,0)$ | 0.206 | -0.209 | -0.254 | 0.318 |
| $(0,-1.8,0)$ | 0.144 | 0.193 | -0.211 | 0.342 |
| $(0,-2.2,0)$ | 0.097 | 0.166 | -0.146 | 0.375 |
| $(0,-2.4,0)$ | 0.080 | 0.151 | -0.110 | 0.387 |
| $(0,-2.8,0)$ | 0.055 | 0.121 | -0.054 | 0.399 |

A strong preference for location of the proton in the $x y$ plane is indicated in Figure 8 which shows valence orbital energies for the equilibrium configuration and two distorted geometries involving displacements of the proton from equilibrium 0.5 au along $x$ and $z$ directions, respectively. In the latter nonplanar case, there is an extensive destabilization of the $6 a_{1} \rightarrow 7 a^{\prime}$ orbital showing that the planar $p$ functions on carbon cannot bond satisfactorily with the proton unless it is in the $x y$ plane as well.

As mentioned previously, face-protonated forms of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$are calculated to be significantly less stable than the optimum side-protonated geometry although the $z=2.0$ and 2.75 au cases reported in Table I are bound relative to $\mathrm{C}_{3} \mathrm{H}_{6}$. Figure 9 shows a plot of valence orbital energies vs. $z$ coordinate of the proton. The sum of orbital energies is lowest at ( $0,0,1.0$ ), but, for the same reasons noted earlier involving a lack of cancellation between nuclear and electronic repulsion, the proton is not bound for this geometry. Of the molecular orbitals available for combination with the proton basis function, only the $3 a_{1}$ interacts significantly, and apparently this orbital is too low lying to bond effec-
tively. A series of additional calculations also supports this point. In Table VI results are reported for $\mathrm{C}_{3} \mathrm{H}_{7}+$ treatments in which molecular orbitals are constrained to be those of $60^{\circ}$ cyclopropane although the Hamiltonian is for $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$. Under such constraints the

Table VI. Results Obtained for Protonated Cyclopropane Using Molecular Orbitals Constrained to be Those of $\mathrm{C}_{3} \mathrm{H}_{6}{ }^{\text {a }}$

| $(x, y, z)$ | $\Delta V_{\mathrm{N}}$ | $\Delta E_{\mathrm{e}}$ | $\Delta E_{\mathrm{T}}$ |
| :---: | :---: | :---: | :---: |
| Face-Protonated Geometry |  |  |  |
| $(0,0,0)$ | 12.702 .57 | -11.93554 | 0.76704 |
| $(0,0,1.0)$ | 11.13906 | -10.73843 | 0.40063 |
| $(0,0,2.0)$ | 8.68906 | -8.60077 | 0.08828 |
| $(0,0,2.5)$ | 7.67287 | -7.63338 | 0.03949 |
| $(0,0,3.0)$ | 6.81470 | -6.79396 | 0.02074 |
| $(0,0,3.5)$ | 6.09338 | -6.07949 | 0.01389 |
| $(0,0,4.0)$ | 5.48697 | -5.47573 | 0.01124 |
| $(0,0,5.0)$ | 4.54219 | -4.53329 | 0.00894 |
|  | Edge-Protonated | $-0 m e t r y$ |  |
| $(0,-0.83,0)$ | 12.60680 | -11.75292 | 0.85388 |
| $(0,-1.23,0)$ | 11.94670 | -11.25986 | 0.68684 |
| $(0,-1.63,0)$ | 10.90718 | -10.48114 | 0.42604 |
| $(0,-1.83,0)$ | 10.33396 | -10.02938 | 0.30458 |
| $(0,-2.43,0)$ | 8.70003 | -8.63534 | 0.06469 |
| $(0,-3.0,0)$ | 7.42892 | -7.44728 | -0.01836 |
| $(0,-3.5,0)$ | 6.53601 | -6.57238 | -0.03637 |
| $(0,-4.0,0)$ | 5.81256 | -5.84859 | -0.03603 |
| $(0,-4.5,0)$ | 5.22092 | -5.25127 | -0.03035 |

${ }^{a} \Delta V_{\mathrm{N}}=$ difference in nuclear repulsion energies of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$ and $\mathrm{C}_{3} \mathrm{H}_{6}, \Delta E_{\mathrm{e}}=$ difference in electronic energies of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$and $\mathrm{C}_{3} \mathrm{H}_{6}$, and $\Delta E_{\mathrm{T}}=$ difference in total energies of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$and $\mathrm{C}_{3} \mathrm{H}_{6}$.
proton never becomes bound in the $\mathrm{C}_{3 \mathrm{v}}$, face-protonated case, but that in the $\mathrm{C}_{2 \mathrm{v}}$ edge-protonated case bonding occurs starting at a $y$ coordinate of -3.0 au . Clearly, such calculations in which no polarization of $\mathrm{C}_{3} \mathrm{H}_{6}$ orbitals is allowed only establish upper bounds to total energies, but the significant point lies in the indication of a first-order difference in bonding ability at the two protonation sites.

## Conclusions

The results of this study show that $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$in the gas phase is stable relative to $\mathrm{C}_{3} \mathrm{H}_{6}$ and that the equilibrium geometry corresponds to that of an edge-protonated species in which the proton is bonded to the ring by a bridged hydrogen bond. Bonding is due principally to an interaction of the proton with in-plane carbon 2 p atomic orbitals of the $3 \mathrm{e}^{\prime} \rightarrow 6 \mathrm{a}_{1}$ molecular orbital of cyclopropane and as such appears preferable to the $2 p_{z}$ bonding above the ring required in the case of a face-protonated species. The proton strongly prefers to remain in the plane of carbon nuclei, and the increase in nuclear repulsion due to the proton is largely responsible for an opening of the ring to approximately $80^{\circ}$ to give maximum stability. Variations in the nuclear and electronic repulsion contributions to the total energy are found to be important factors in determining the equilibrium geometry of this system, consequently ruling out the possibility of quantitative geometry predictions based solely on orbital energy calculations.

The question of rearrangement of $\mathrm{C}_{3} \mathrm{H}_{7}^{+}$to give methyl cation and ethylene has not been explored explicitly in this study, but previous calculations on these systems ${ }^{12,15}$ using comparable basis sets indicate that $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$is slightly lower in energy than the sum of ground-state energies of $\mathrm{CH}_{3}{ }^{+}$and $\mathrm{C}_{2} \mathrm{H}_{4}$. Addition of an electron to give $\mathrm{CH}_{3}$ and $\mathrm{C}_{2} \mathrm{H}_{4}$ is highly favorable energetically, however. The calculated edge-protonated geometry of $\mathrm{C}_{3} \mathrm{H}_{7}{ }^{+}$is in agreement with the proposed bridged hydrogen structure of protonated cyclopane intermediates in solution.

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[^0]:    (18) All energies and coordinates given in this paper are in atomic units, 27.21 eV and $0.529 \AA$, respectively.

